B. Sc. Part-III

Paper-V, Group A

Problem set on partial differential equations —

Q.1 Consider the one-dimensional heat equation $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\sigma}\frac{\partial T}{\partial t}$.

If α is the separation constant, discuss the nature of the solution for

(a) $\alpha^2 > 0$, (b) $\alpha^2 = 0$, (c) $\alpha^2 < 0$.

(d) Find the particular solution for

$T(0,t) = T(L,t) = 0$ and $T(x,0) = T_0 \sin\frac{n\pi x}{L}$ (for $0<x<L$)

if the general solution is

$$T(x,t) = C e^{-\alpha^2 \sigma t}(A\cos\alpha x + B\sin\alpha x).$$

Q.2 The motion of a transverse wave in a string is characterized by $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$

(i) Discuss the nature of the solution for $k^2>0$, $k^2=0$, $k^2<0$, where k^2 is the separation constant.

(ii) Find the general periodic motion solution of the wave equation.

(iii) For $\psi(0,t) = \psi(L,t) = \left.\frac{\partial \psi}{\partial t}\right|_{t=0} = 0$ and $\psi(x,0) = Y$

show that the solution for periodic motion is

$$\psi(x,t) = \sum_{n=1}^{\infty} Y_n \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n v \pi t}{L}\right).$$

Q.3 Consider the two-dimensional Laplace equation,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0.$$

(i) Show that $\psi(x,y) = (A\cos\alpha x + B\sin\alpha x)(C\cosh\alpha y + D\sinh\alpha y)$

where α^2 is the separation constant.

(ii) For $\psi(x,0) = \psi(0,y) = \psi(w,y) = 0$ and $\psi(x,b) = \psi_0 \sin\frac{n\pi x}{w}$

find $\psi(x,y)$.